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A1_6 The Ultimate Curveball

O. Hophuong, L. Li, L. Warford, T. Wu

Department of Physics and Astronomy, University of Leicester, Leicester, LE1 7RH

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Abstract

Drag effects play an important effect in everyday life, but is most noticeable in ball games such as baseball and football. In this paper, we determine the initial velocity and rotational velocities required for a ball to come back to the thrower. This is done by considering the Magnus effect. We find that these properties are proportional to each other, but the relationship between the object's mass and radius plays a bigger role than expected.

Introduction

The Magnus force is a drag force, resulting due to the object's spin. Air moves faster on one side than the other, allowing for seemingly gravity defying tricks. By considering the drag forces on the particle and obtaining equations of motion, we can find the required initial conditions to throw a ball and have it return as a result of its spin.

Theory

The Magnus force acts in the direction perpendicular to the velocity and the axis of rotation of the ball, with magnitude [1]:

$$F_M = \frac{4}{3}(4\pi^2 {r_b}^3 \omega_b \rho_{air} v) \tag{1}$$

where r_b is the radius of the ball, ω_b is the rotational velocity of the ball (or, its spin) and v is the velocity of the ball. We consider the scenario in which we pitch the ball parallel to the xy plane. We assume that the Magnus (and drag) force in the z-direction is negligible compared to the force of gravity so that we can consider the z direction and the xy plane separately.

First, we consider the deceleration of the ball as it travels around in the xy plane, which is just the drag force. From Newton's Second Law,

$$\frac{1}{2}\rho_{air} v^2 C_D A = ma \tag{2}$$

where C_D is the drag coefficient and A is the cross-sectional area. Re-arranging this gives us a differential equation in v:

$$\beta v^2 = \frac{dv}{dt}, \text{where } \beta = \frac{\rho_{air} C_D \pi {r_b}^2}{2m} \tag{3}$$

Solving Eq. (3) and substituting for our initial conditions, i.e. at $t = 0$, $v = v_0$:

$$v(t) = -\frac{1}{\beta t - \frac{1}{v_0}} \tag{4}$$

We assume ω_b is constant with time, i.e. it does not decay. The Magnus force acts as a centripetal force, so using Eqs. (1) and (4):

$$\omega_b = \frac{1}{\gamma r(t)} \frac{1}{\beta t - \frac{1}{v_0}}, \text{where } \gamma = \frac{16\pi^2 {r_b}^3 \rho_{air}}{3m} \tag{5}$$

From the equations of motion with constant acceleration, we have $t = \sqrt{\frac{2h}{g}}$, where h is the height over which the ball falls, giving:

$$\omega_b = \frac{1}{\gamma r(t)} \left[\frac{1}{\beta \sqrt{\frac{2h}{g}} - \frac{1}{v_0}} \right] = \frac{A}{\gamma r(t)} \qquad (6)$$

$r(t)$ gives the radius from centre of rotation of the ball's motion, which is an outwards spiral. We assume that the point about which the motion is circular, is constant and take it as our origin.

The ball is caught once it is within arm's reach. This can be expressed by equating $r(t)$ to the equation of the off center circle, centred on us.

$$r(t)^2 = {r_c}^2 - r(0)^2 - 2r(t)r(0)\cos(\theta - \phi)[2] \quad (7)$$

where r_c is the radius in which we can catch the ball, $r(0) = \frac{v_0}{\gamma \omega_b}$ from Eq. (6) and taking $t = 0$, θ is the angle from the origin to some point on the circle centred around us and ϕ is the angle from the origin to the us, the center of the circle. For simplicity, take $\phi = 0$. Solving Eq. (8) and simplifying:

$$r(t) = r(0)\cos\theta \pm \sqrt{{r_c}^2 - r(0)^2 \sin^2\theta} \qquad (8)$$

where we have 2 values due to the symmetry of $\cos\theta$. Substituting into Eq. (7):

$$\omega_b = \pm \frac{1}{r_c \gamma} \sqrt{A^2 + {v_0}^2 - 2Av_0 \cos\theta} \qquad (9)$$

These two values correspond to the ball moving clockwise and counter-clockwise, which in turn corresponds to the two possible values of $r(t)$.

Results

The parameters that are not associated with the ball can be held constant: $\rho_{air} = 1.225$ kg m^{-3} [3], $h = 1$ m (so that we can catch the ball) and $r_c = 0.5$ m so that the catch is easy.

However, we can try different values for the parameters of the ball. Taking values for a baseball, a tennis ball [5] and a football [6], assuming that velocities are high enough to be in turbulent flow, i.e. $C_D = 0.47$ [7], we produce the following plot. Note that changing θ did not appear to make a large difference to the shape of the plot.

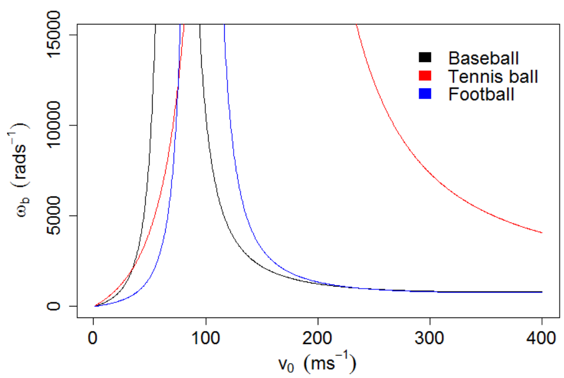

Figure 1: Plots of ω_b against v_b for a baseball ($r_b = 0.036$ m, $m = 0.035$ kg) a tennis ball ($r_b = 0.033$ m, $m = 0.056$ kg) and a football ($r_b = 0.11$ m, $m = 0.43$ kg).

Discussion and Conclusion

For reasonable speeds, an increase in v_0 corresponds to an increase in ω_b, as expected. However, at high v_0 and ω_b, we have an unexpected peak; at these speeds our assumption of a constant point of rotation breaks down. A calculation of $r(0)$ in this scenario gives a value of order 10^{-1} m, confirming this.

Even in the low speed regime, we find that despite the similar masses of the baseball and tennis balls, the change in radius makes a significant difference to the shape of the plot. This mass-radius relationship could be explored in a future paper.

References

[1] https://goo.gl/zu4En2 [Accessed 29. Nov 2017]

[2] https://goo.gl/m33zGP [Accessed 13. Dec 2017]

[3] https://goo.gl/Gu33Cw [Accessed 29. Nov 2017]

[4] https://goo.gl/7NYbjD [Accessed 29. Nov 2017]

[5] https://goo.gl/8UbKvn [Accessed 29. Nov 2017]

[6] https://goo.gl/LYi2Fc [Accessed 29. Nov 2017]

[7] https://goo.gl/ZVw942 [Accessed 29. Nov 2017]